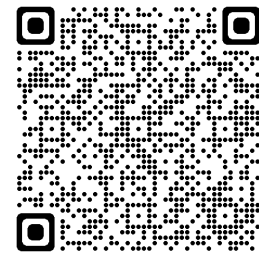


Reinsurance Decision Making: Past, Present and Future



John A. Major, ASA
Stephen J. Mildenhall

CAS Seminar on Reinsurance, June 2022

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Donald F. Mango 1963-2022



INSURANCE CAPITAL AS A SHARED ASSET
BY
DONALD MANGO

ASTIN BULLETIN, Vol. 35, No. 2, 2005, pp. 471-486

Capital Tranching: A RAROC Approach to Assessing Reinsurance Cost Effectiveness

by Donald Mango, John Major, Avraham Adler, and Claude Bunick

CASUALTY ACTUARIAL SOCIETY

VOLUME 7/ISSUE 1

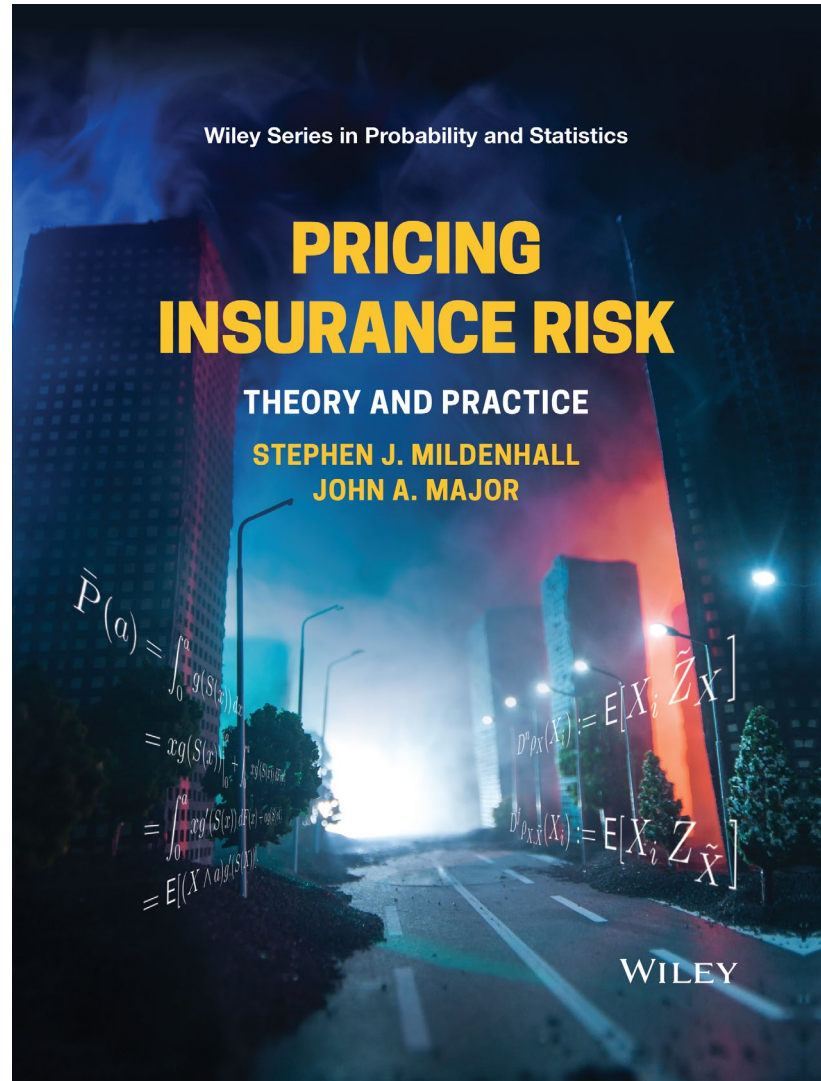


convex risk

Reinsurance Decision Making: Past, Present and Future Part I: Pricing with Spectral Risk Measures

Stephen J. Mildenhall

CAS Seminar on Reinsurance, June 2022





Portfolio Pricing: Five Simplifying Assumptions and One Objective

1. One-period model

2. No existing business

3. No taxes

4. No investment income

5. No expenses (handled separately)

Load loss cost for risk

Caveat: Everything presented is true most of the time, nothing is true all the time.



Pricing Functional: The Idea

Stand-alone risk \rightarrow Premium

$$X \rightarrow \rho(X)$$

X = random variable of outcomes

- Cat model
- Casualty simulation model
- Parametric distribution



Pricing Functional: Desirable Properties

- Consistent with prices in a competitive market

- 1. Monotone: $X \leq Y$ implies $\rho(X) \leq \rho(Y)$

- 2. Respects diversification: $\rho(X + Y) \leq \rho(X) + \rho(Y)$

- 3. But...no credit when no diversification
 - If outcomes X and Y imply same event order, then $\rho(X + Y) = \rho(X) + \rho(Y)$

- 4. $\rho(X)$ only depends on the distribution of X

- Jargon: 2 = sub-additive, 3 = comonotonic additive, 4 = law invariant (**SCALI**)



SCALI Properties Define a Spectral Risk Measure (SRM)

- SRMs have **four** different representations of $\rho(X)$
 1. Weighted average of VaRs
 2. Weighted average of TVaRs
 3. Worst over a set of probability scenarios
 4. Distorted expected value
- Distorted expected value: there exists an increasing, concave distortion function g so that

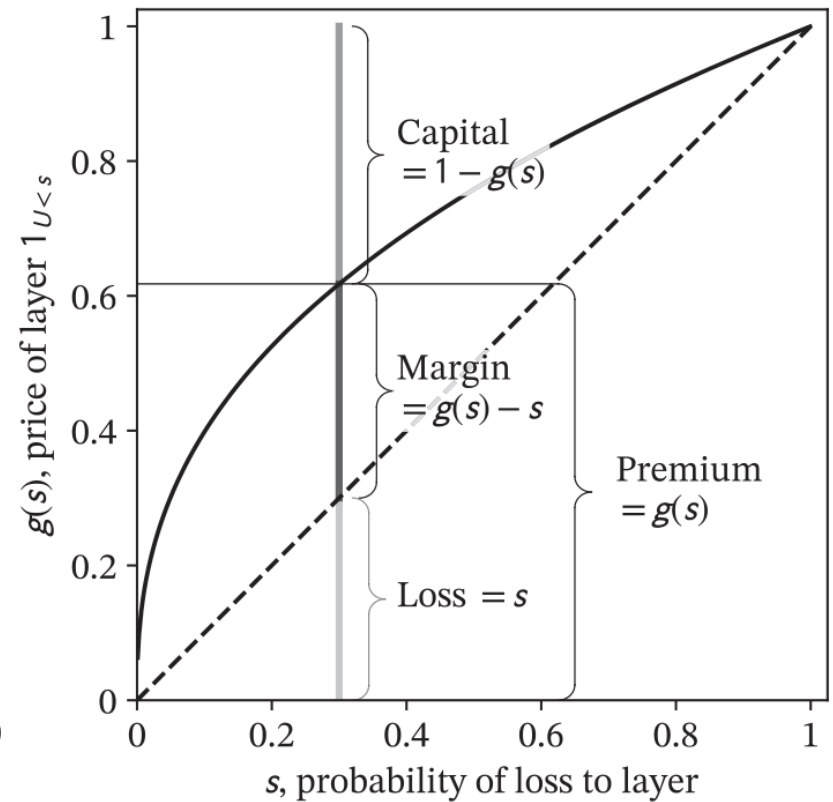
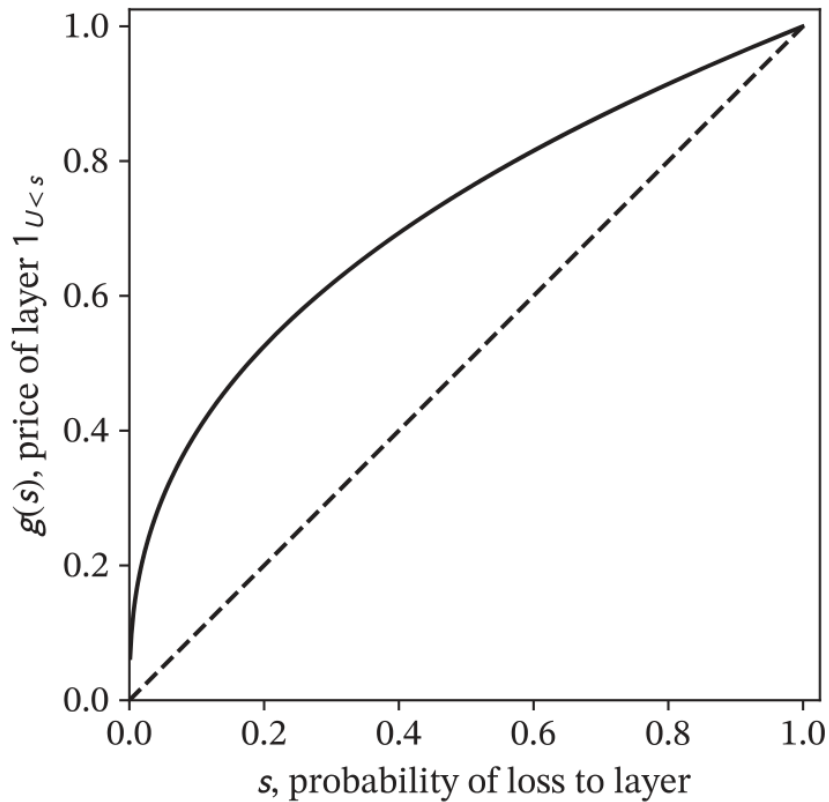
$$\rho_g(X) := \int_0^\infty g(S_X(x)) dx \quad \Big| \quad = \mathbb{E}[Xg'(S(X))]$$

where $S_X(x) = \Pr(X > x)$ is the survival function of X

- Expectation representation shows SRMs have a **natural allocation** $\mathbb{E}[X_i g'(S(X))]$, which also equals the marginal allocation



Distortion Function g Prices Bernoulli 0/1 Risk





Spectral Risk Measure Portfolio Pricing

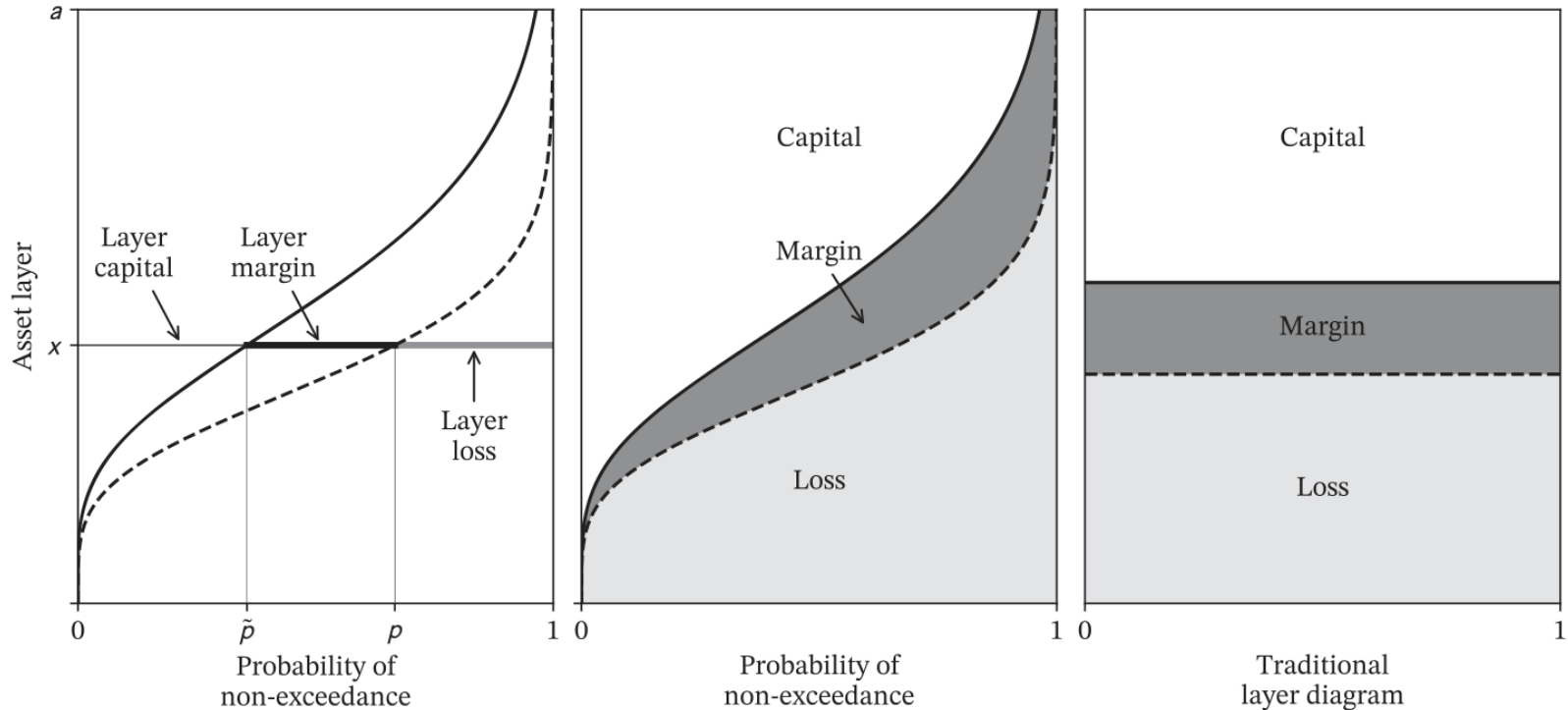


Figure 10.5 Continuum of risk sharing varying by layer of loss (dashed) and premium (solid): by layer (left), in total summed over layers (middle), and traditional (right). The total loss, margin, and capital areas are equal in the middle and right plots. Losses in the left and middle plots are Lee diagrams.

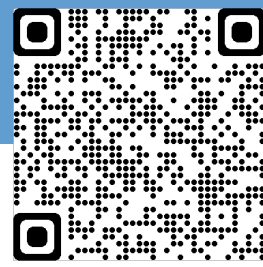


Case Study: Financial Model

- InsCo. has only **two sources** of assets
 - **Policyholders** pay **premium** by buying policies at InsCo's asking price
 - **Investors** contribute **capital** by buying residual value at their bid price

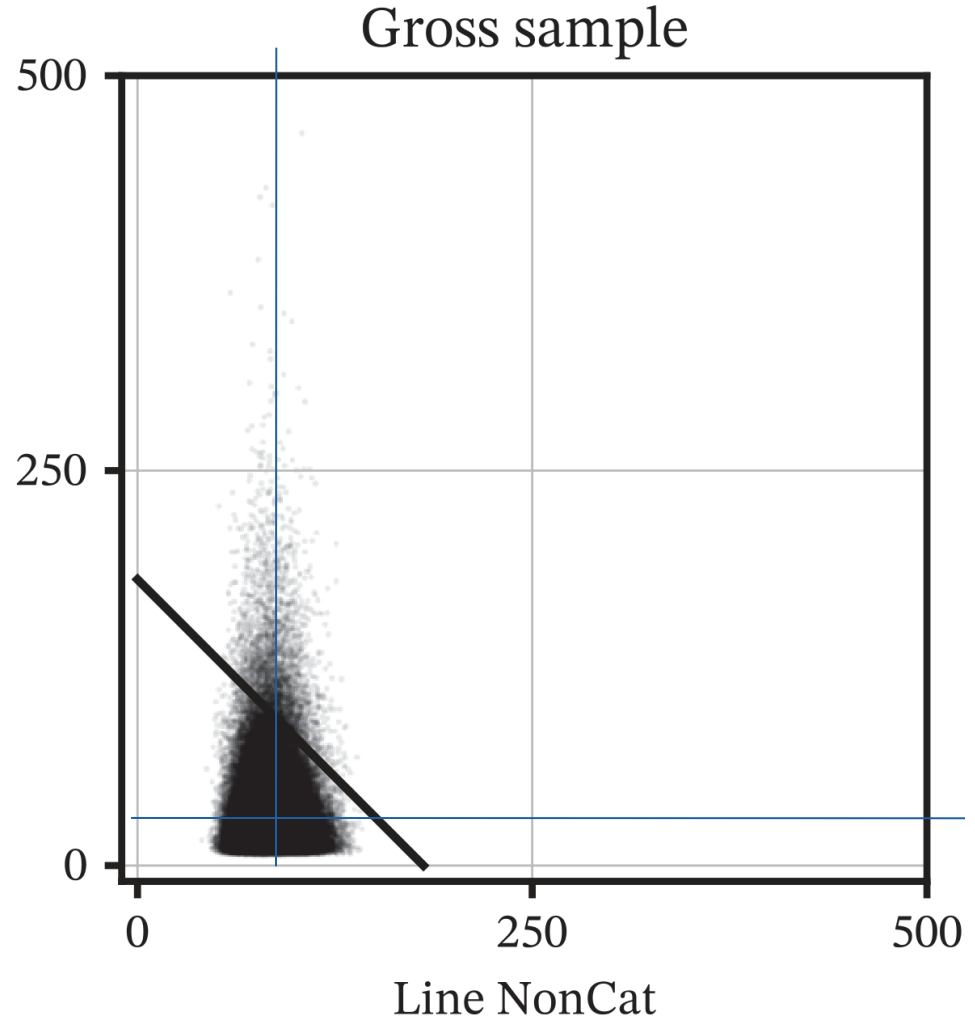
- At time 0
 - Premium P
 - Capital Q
 - Assets $a = P + Q$
 - Asset amount a is set by regulator/rating agency

- At time 1
 - Claims X revealed
 - Policyholder payments $X \wedge a = \min(X, a)$
 - Investor return $(a - X)^+ = \max(0, a - X)$



Case Study: Cat/NonCat Stochastic Model

- NonCat: gamma
 - mean 80, cv 0.15
- Cat: lognormal
 - mean 20, cv 1.0
- Independent
- Total
 - mean 100, cv 0.233
- Asset requirement
 - VaR 99.9% = 267.2





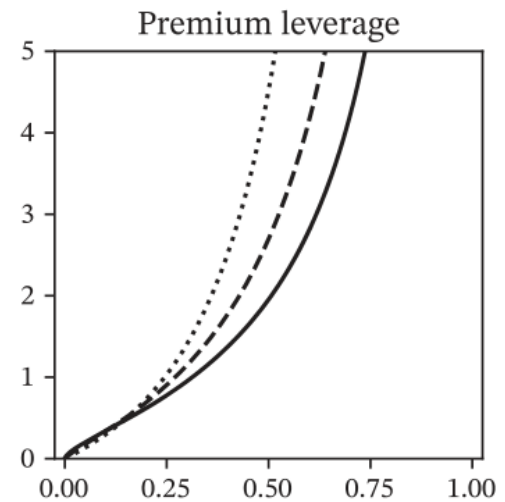
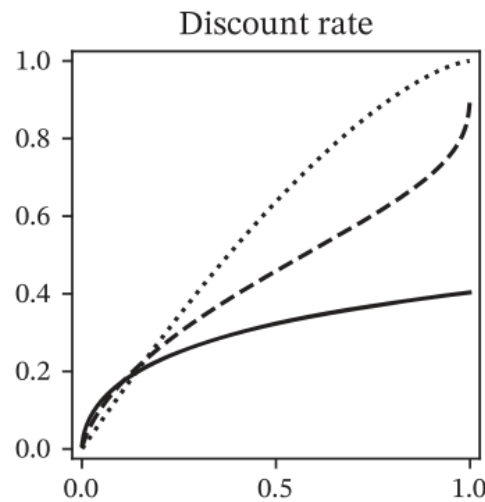
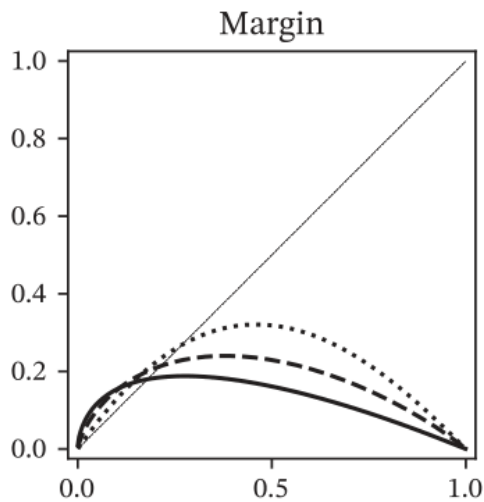
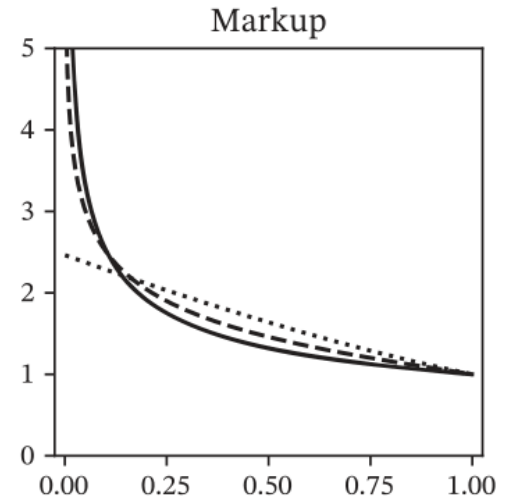
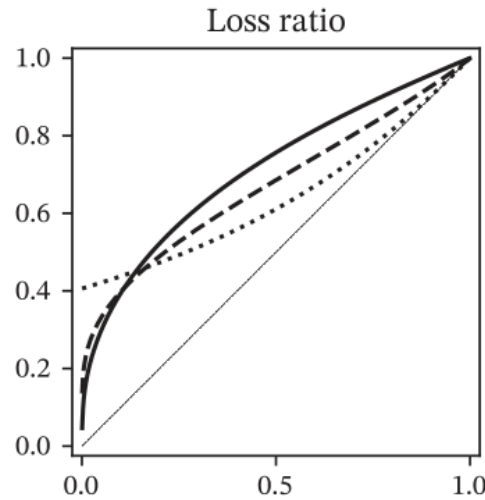
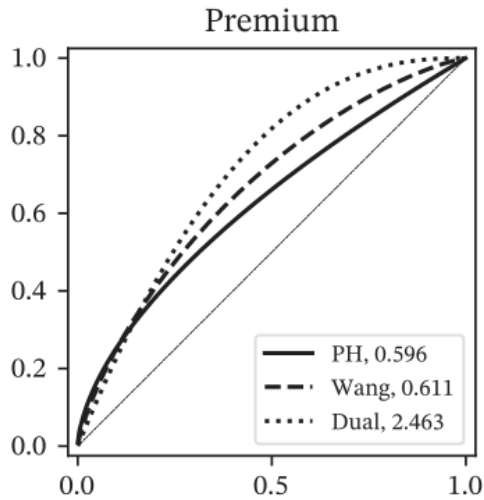
Example g, Shape of g and Properties of ρ

Table 11.5 Parameters for the six SRMs and associated distortions that are applied in Section 11.4.

ID	Distortion	$g(s)$	Tame	Cat/Non-Cat	Hu/SCS
CCoC	CCoC	$(s + \iota)/(1 + \iota)$	$\iota = 0.10$	0.10	0.10
PH	Proportional hazard	s^α	$\alpha = 0.683$	0.596	0.449
Wang	Wang	$\Phi(\Phi^{-1}(s) + \lambda)$	$\lambda = 0.375$	0.611	1.190
Dual	Dual moment	$1 - (1 - s)^m, m \geq 1$	$m = 1.576$	2.463	12.029
TVaR	TVaR	$s/(1 - p) \wedge 1$	$p = 0.227$	0.482	0.899
Blend	PWL				

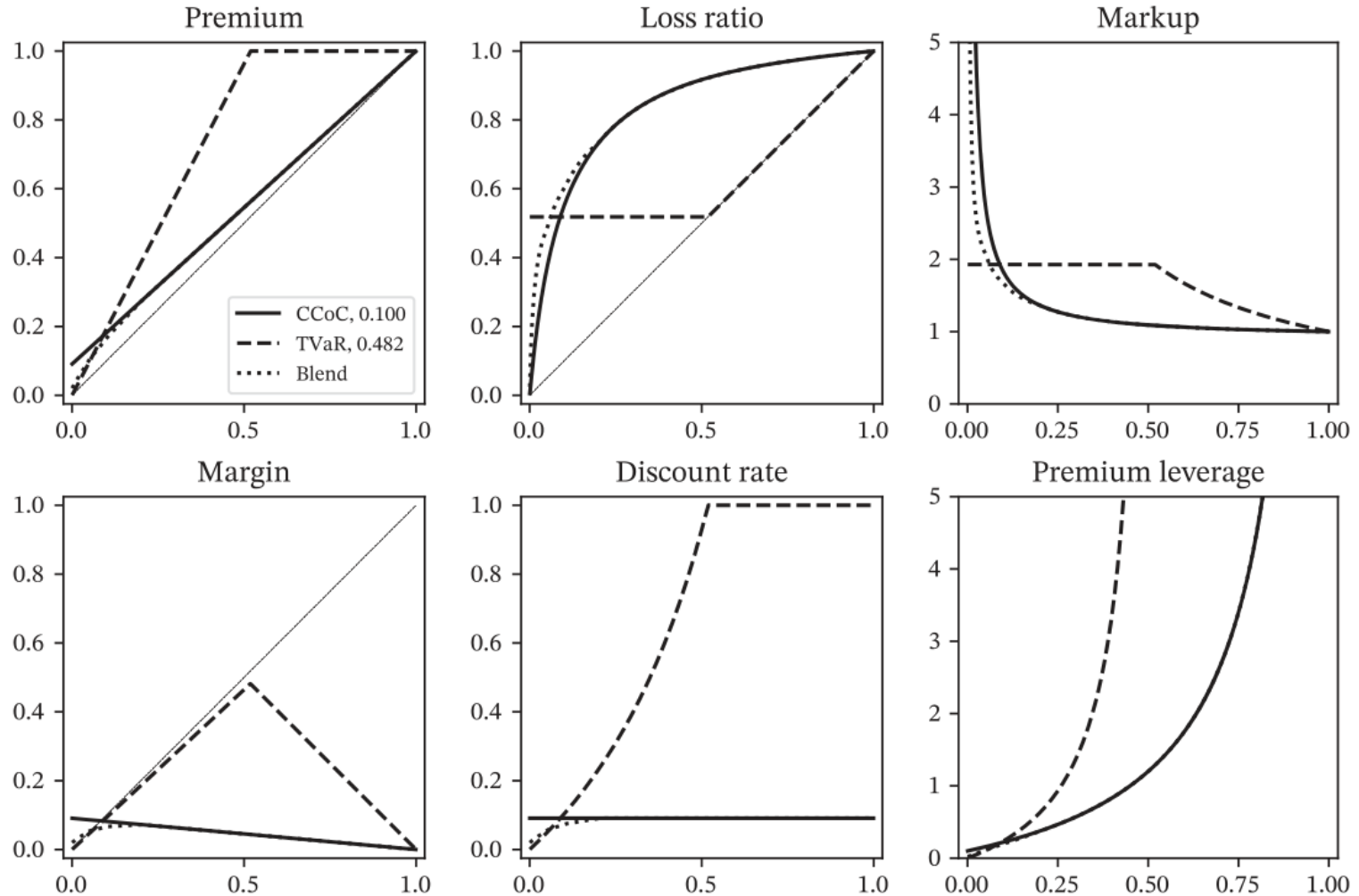


Shape of g and Properties of ρ





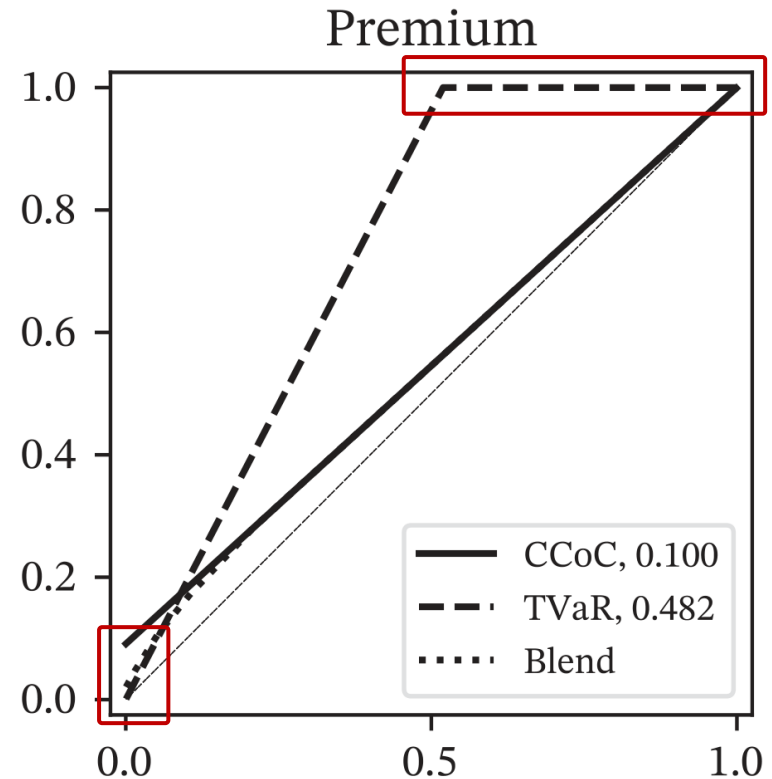
Shape of g and Properties of ρ





Shape of g and Properties of ρ

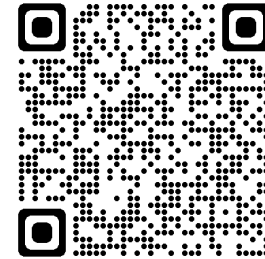
1. If g is **steep near $s=0$** it has expensive tail-risk capital
 - $\text{CCoC} > \text{PH} > \text{Wang} > \text{Dual} > \text{TVaR}$
2. If g is **flat near $s=1$** it has expensive body-risk capital
 - Opposite order
3. CCoC vertical at 0: has the most expensive tail-risk and cheapest body-risk capital
4. TVaR flat at 1: has the most expensive body-risk and cheapest tail-risk capital





Stand-Alone Pricing: Cat low 30s, NonCat upper 80s

Statistic	Distortion	Gross				Net		
		Cat	Non-Cat	SoP	Total	Cat	SoP	Total
Loss	Blend	19.95	80.00	99.94	99.95	17.73	97.73	97.73
Margin	CCoC	15.03	3.84	18.87	15.21	7.99	11.83	8.17
	PH	13.81	6.37	20.17	15.21	7.38	13.75	9.82
	Wang	12.88	7.51	20.38	15.21	7.95	15.45	11.06
	Dual	11.88	8.6	20.48	15.21	8.83	17.43	12.40
	TVaR	11.21	9.17	20.38	15.21	9.15	18.32	13.15
	Blend	6.49	2.39	8.88	6.81	3.44	5.83	4.09
	Premium	CCoC	34.98	83.84	118.8	115.2	25.72	109.6
PH		33.75	86.36	120.1	115.2	25.11	111.5	107.6
Wang		32.82	87.50	120.3	115.2	25.68	113.2	108.8
Dual		31.83	88.59	120.4	115.2	26.57	115.2	110.1
TVaR		31.15	89.17	120.3	115.2	26.88	116.0	110.9
Blend		26.43	82.39	108.8	106.8	21.17	103.6	101.8
Loss ratio	CCoC	0.57	0.954	0.841	0.868	0.689	0.892	0.923
	PH	0.591	0.926	0.832	0.868	0.706	0.877	0.909
	Wang	0.608	0.914	0.831	0.868	0.691	0.863	0.898
	Dual	0.627	0.903	0.83	0.868	0.667	0.849	0.887
	TVaR	0.64	0.897	0.831	0.868	0.66	0.842	0.881
	Blend	0.755	0.971	0.918	0.936	0.838	0.944	0.96





Allocated Pricing using the Natural Allocation

Statistic	Method	Gross			Net			Ceded			
		Cat	Non-Cat	Total	Cat	Non-Cat	Total	Diff			
Loss	Expected loss	19.96	79.99	99.95	17.75	79.98	97.73	2.21			
Margin	Expected loss	3.04	12.17	15.21	1.78	8.04	9.82	5.38			
	Dist ROE	22.02	-6.82	15.21	13.94	-5.77	8.17	7.04			
	Dist PH	12.94	2.27	15.21	6.32	3.51	9.82	5.38			
	Dist Wang	11.31	3.9	15.21	6.15	4.91	11.06	4.15			
	Dist dual	9.72	5.49	15.21	6.47	5.94	12.40	2.8			
	Dist TVaR	8.74	6.47	15.21	6.65	6.5	13.15	2.05			
	Dist blend	7.32	-0.507	6.81	4.31	-0.217	4.09	2.73			
Stand-Alone Premium	Expected loss	22.99	92.16	115.2	19.53	88.02	107.6	7.6			
Premium	CCoC	34.98	83.84	Dist ROE	41.98	73.17	115.2	31.68	74.22	105.9	9.25
	PH	33.75	86.36	Dist PH	32.89	82.26	115.2	24.06	83.49	107.6	7.6
	Wang	32.82	87.50	Dist Wang	31.26	83.89	115.2	23.89	84.89	108.8	6.36
	Dual	31.83	88.59	Dist dual	29.68	85.48	115.2	24.21	85.92	110.1	5.02
	TVaR	31.15	89.17	Dist TVaR	28.69	86.46	115.2	24.39	86.49	110.9	4.27
	Blend	26.43	82.39	Dist blend	27.28	79.48	106.8	22.05	79.77	101.8	4.94
	Loss ratio	Expected loss	0.868	0.868	0.868	0.909	0.909	0.909	0.909	0.291	
	Dist ROE	0.475	1.09	0.868	0.560	1.08	0.923	0.239			
	Dist PH	0.607	0.972	0.868	0.738	0.958	0.909	0.291			
	Dist Wang	0.638	0.954	0.868	0.743	0.942	0.898	0.348			
	Dist dual	0.673	0.936	0.868	0.733	0.931	0.887	0.441			
	Dist TVaR	0.696	0.925	0.868	0.728	0.925	0.881	0.519			
	Dist blend	0.732	1.01	0.936	0.805	1	0.96	0.448			

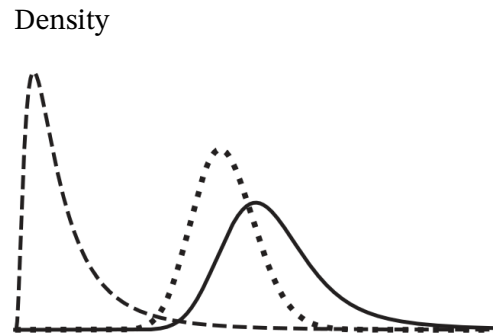
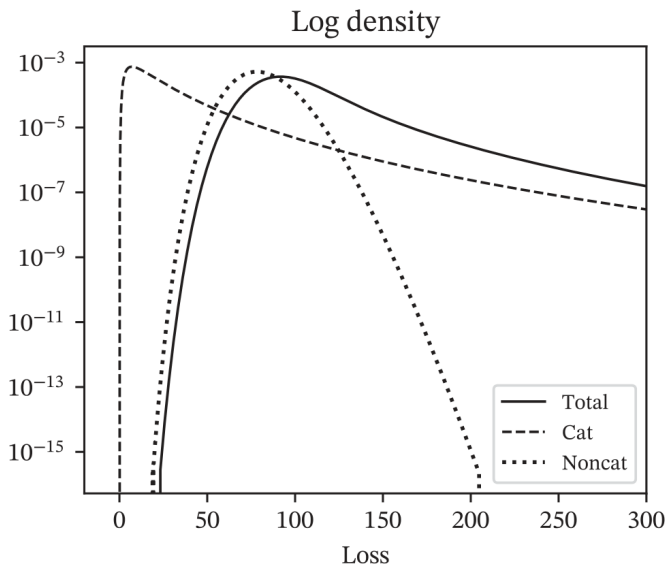
Source: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley [CORRECTED]



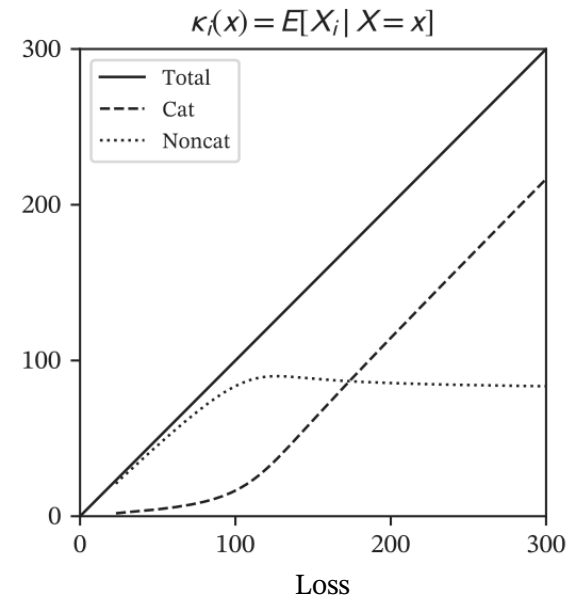
Why Do the Allocations Make Sense?

	Cat	Total	NonCat
	PH, Tail-Centric		
Stand-alone	33.75	120.11	86.36
Allocation	32.89	115.15	82.26
Change	0.86	← 4.96 →	4.1
Pct	17%		83%

	Cat	Total	NonCat
	TVaR, Body-Centric		
Stand-alone	31.15	120.32	89.17
Allocation	28.69	115.15	86.46
Change	2.46	← 5.17 →	2.71
Pct	48%		52%



- Cat: thick tail, narrow body
- NonCat: thin tail, broad body



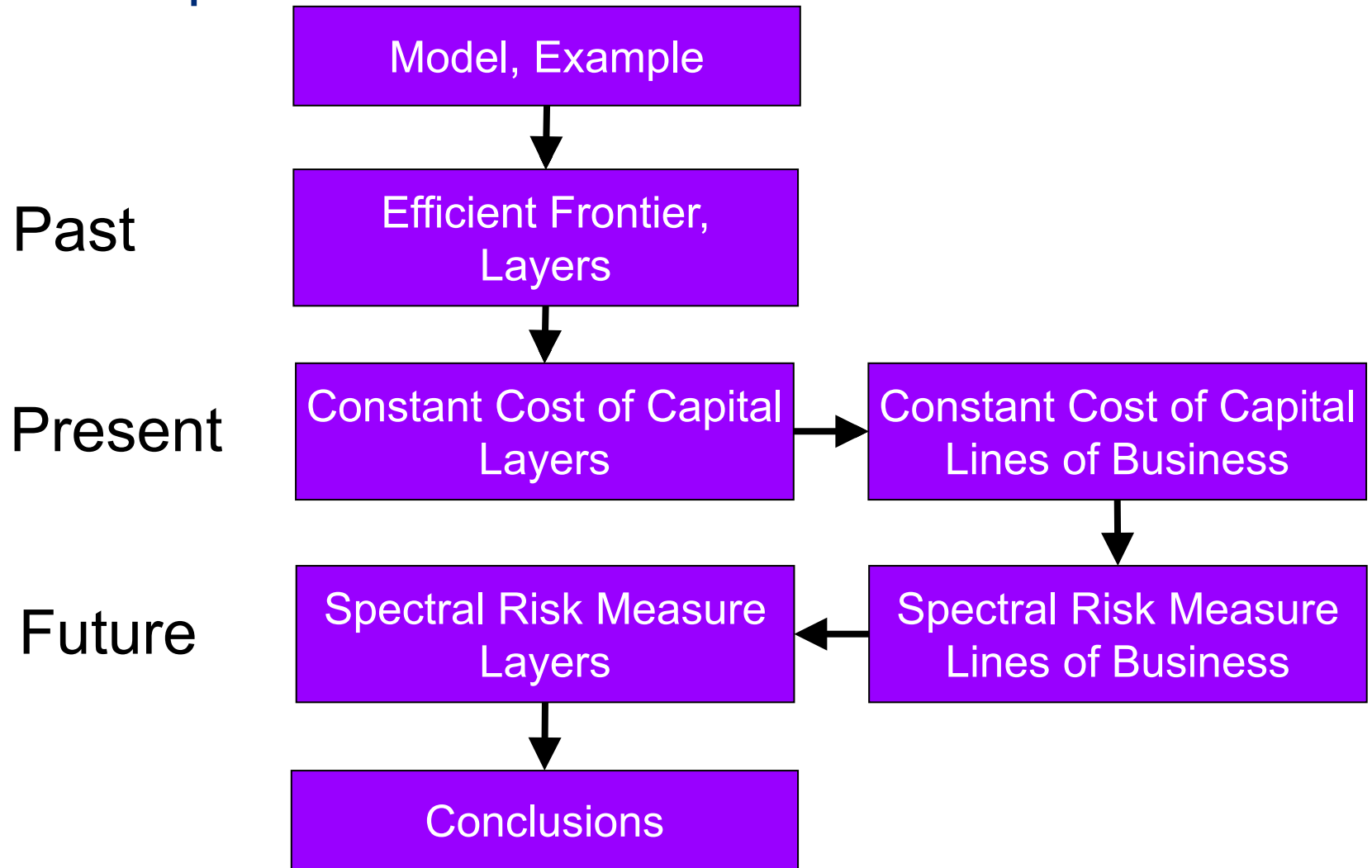
Reinsurance Decision Making: Past, Present and Future

Part 2: Reinsurance Applications

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CAS Seminar on Reinsurance, June 2022

Roadmap



Adding Reinsurance to the Financial Model

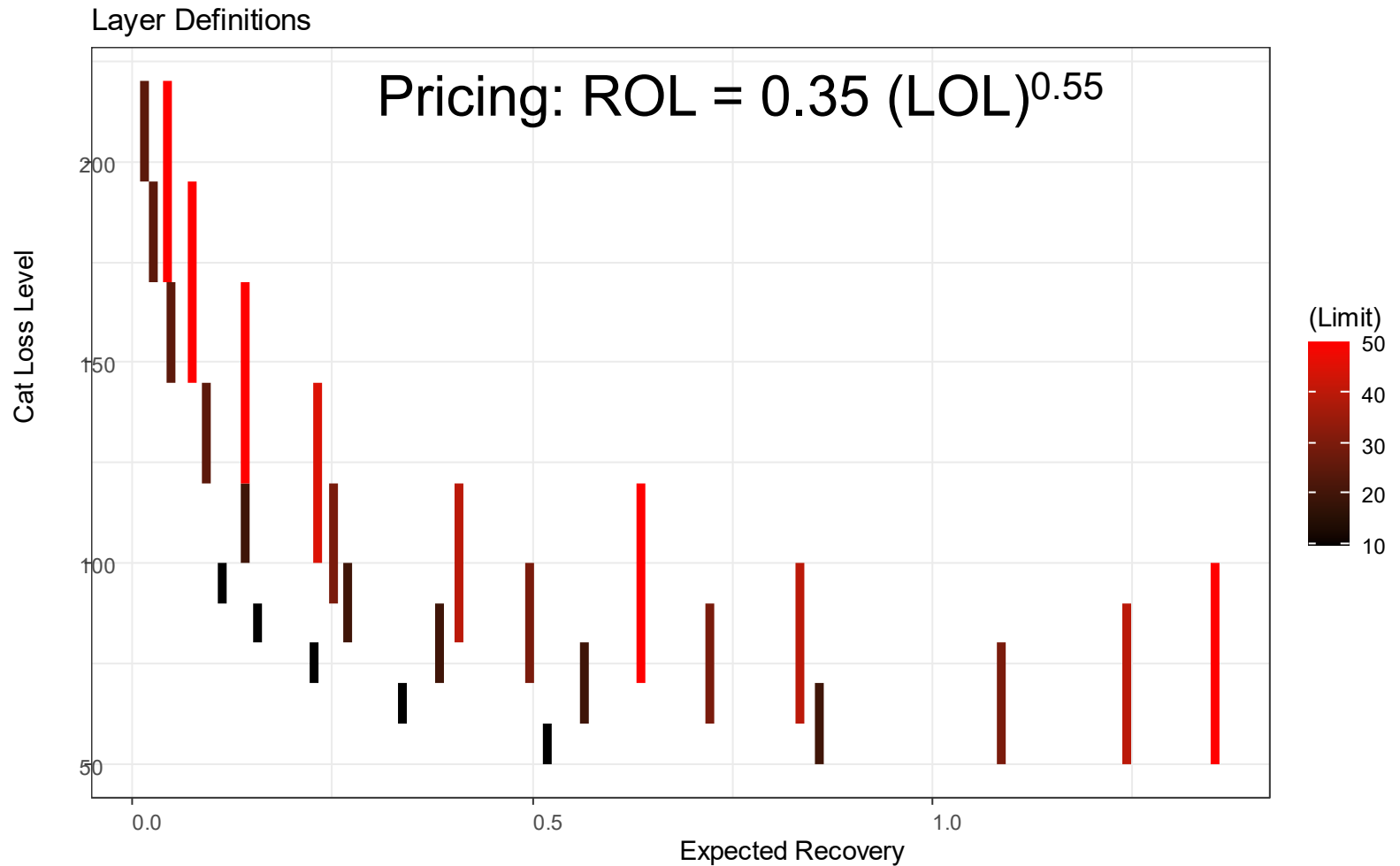
- **Three** sources of assets
 - Policyholders P_{Re}
 - Investors Q_n
 - Reinsurance X_c (contingent ceded losses)
- Gross loss $X = X_n + X_c$
- Cost of reinsurance (ceded premium) π
- $P_{Re} = P_n + \pi$, net premium plus reinsurance premium
- Capital requirement
 - $a_n = P_n + Q_n$, net asset requirement
 - $\Delta a = a - a_n$, capital benefit from reinsurance

Example Cat/NonCat Portfolio

- Non-cat: Gamma mean 80, cv 0.15
- Cat: Lognormal mean 20, cv 1.0
- Independent
- Total mean 100, cv 0.233

- **Asset requirement**
 - **VaR 99.8% = 237.5**
- **Target return 6%**

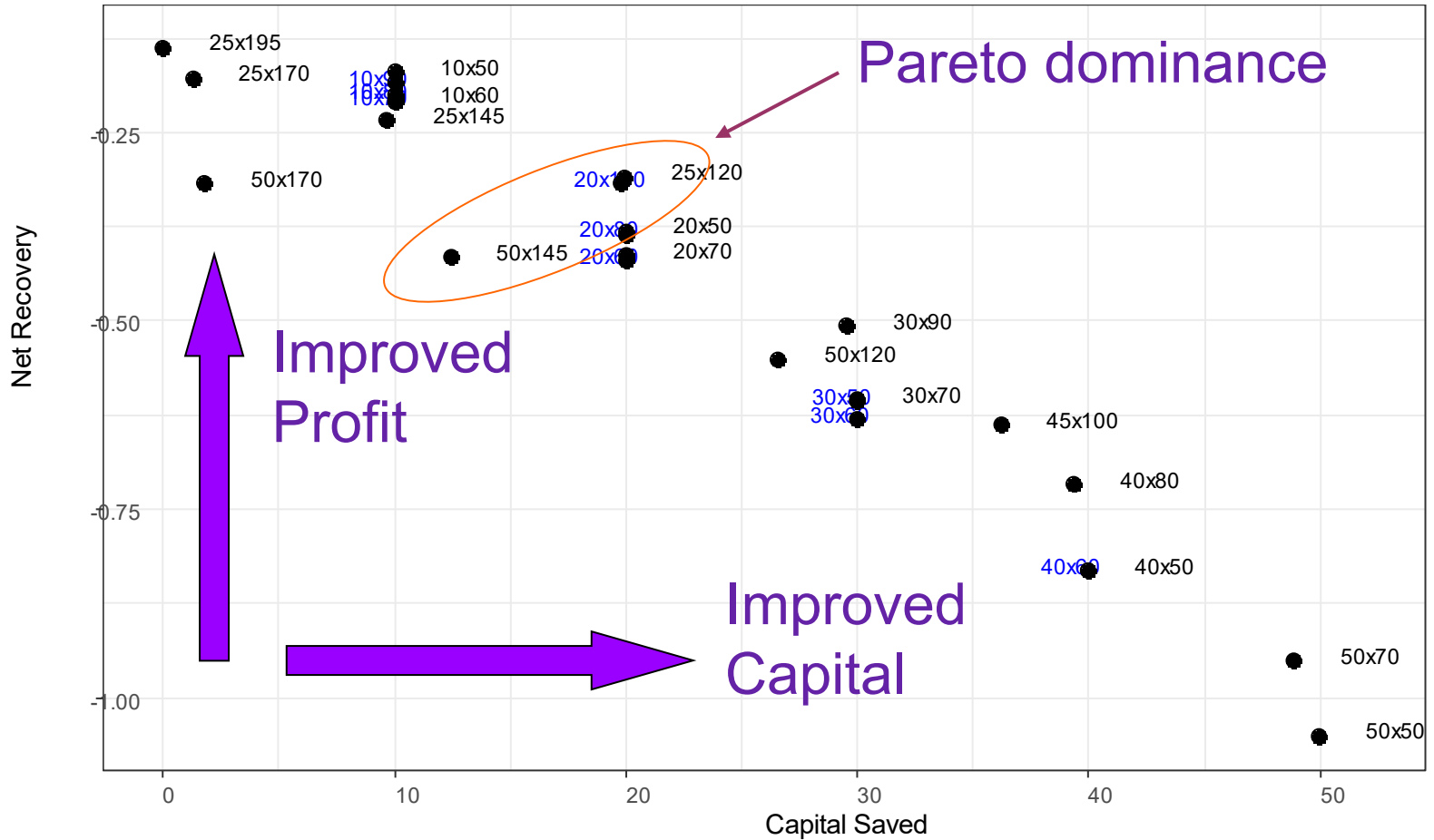
Reinsurance Cat Agg XOL Layers



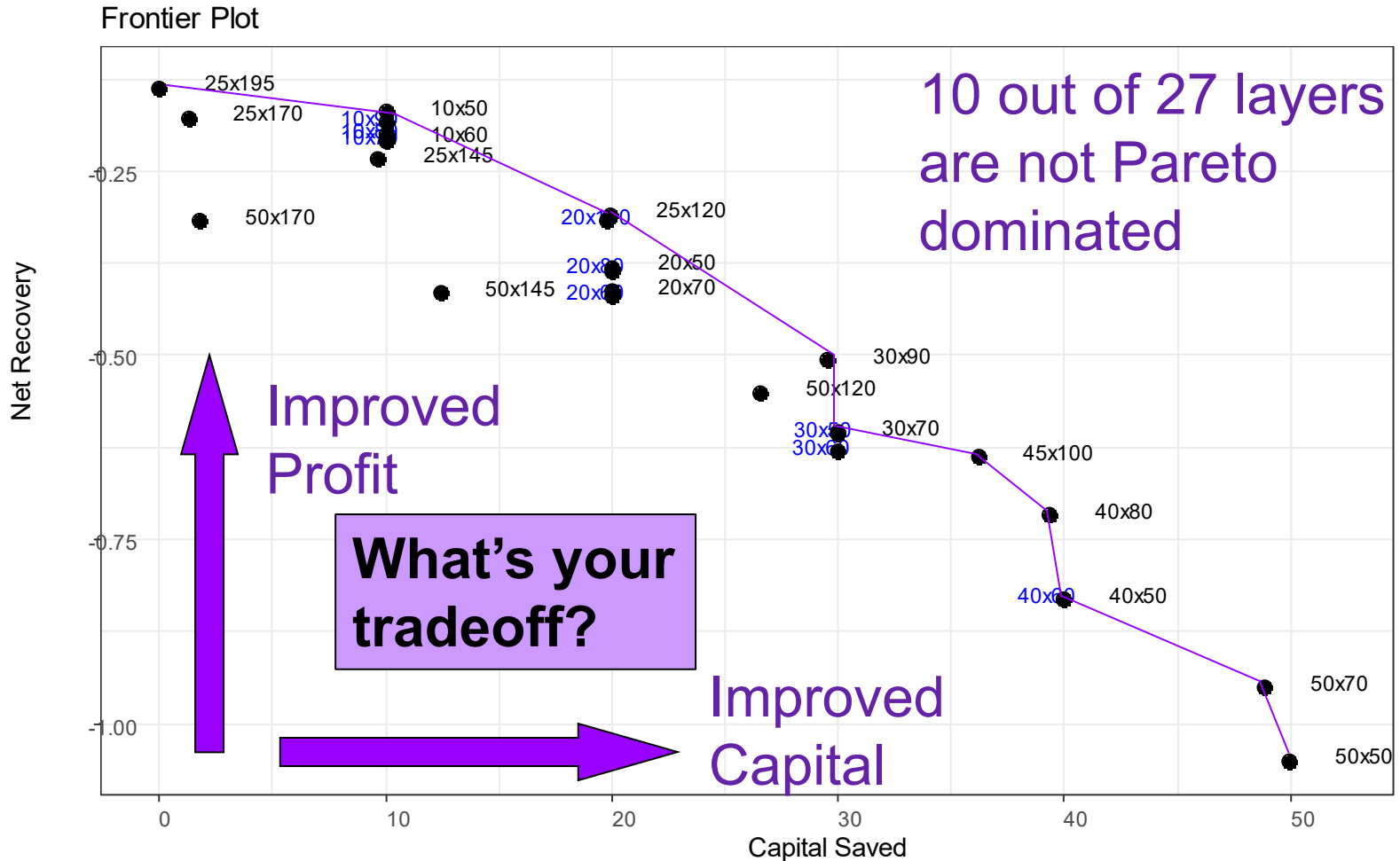
Decisions Past: Multiple Criteria

- Goals:
 - Maximize net recovery $E[X_c] - \pi$
 - Maximize capital savings Δa

Frontier Plot

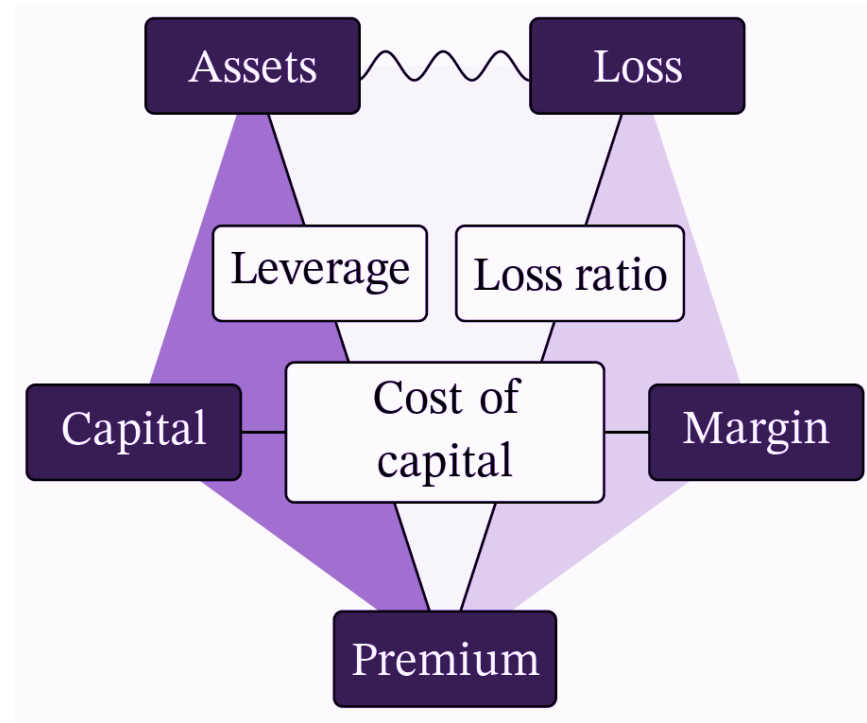


So where do you want to be?



Decisions Present: The Portfolio Cost of Capital

- Basic logic:
 - Policyholder premiums $P = EL + M$
 - Investor capital $Q = a - P$
 - Expected return $\iota = M / Q$
- Conclusion
 - $P = EL + \iota (a - P)$
 - $= (EL + \iota a) / (1 + \iota)$
 - $= v EL + d a$ ←
- Note
 - $v = 1 / (1 + \iota)$ is the risk discount factor
 - $d = \iota / (1 + \iota) = \iota v$ is the rate of risk discount
 - $v + d = 1$



Impact of Reinsurance on Premium

- Funded with capital only

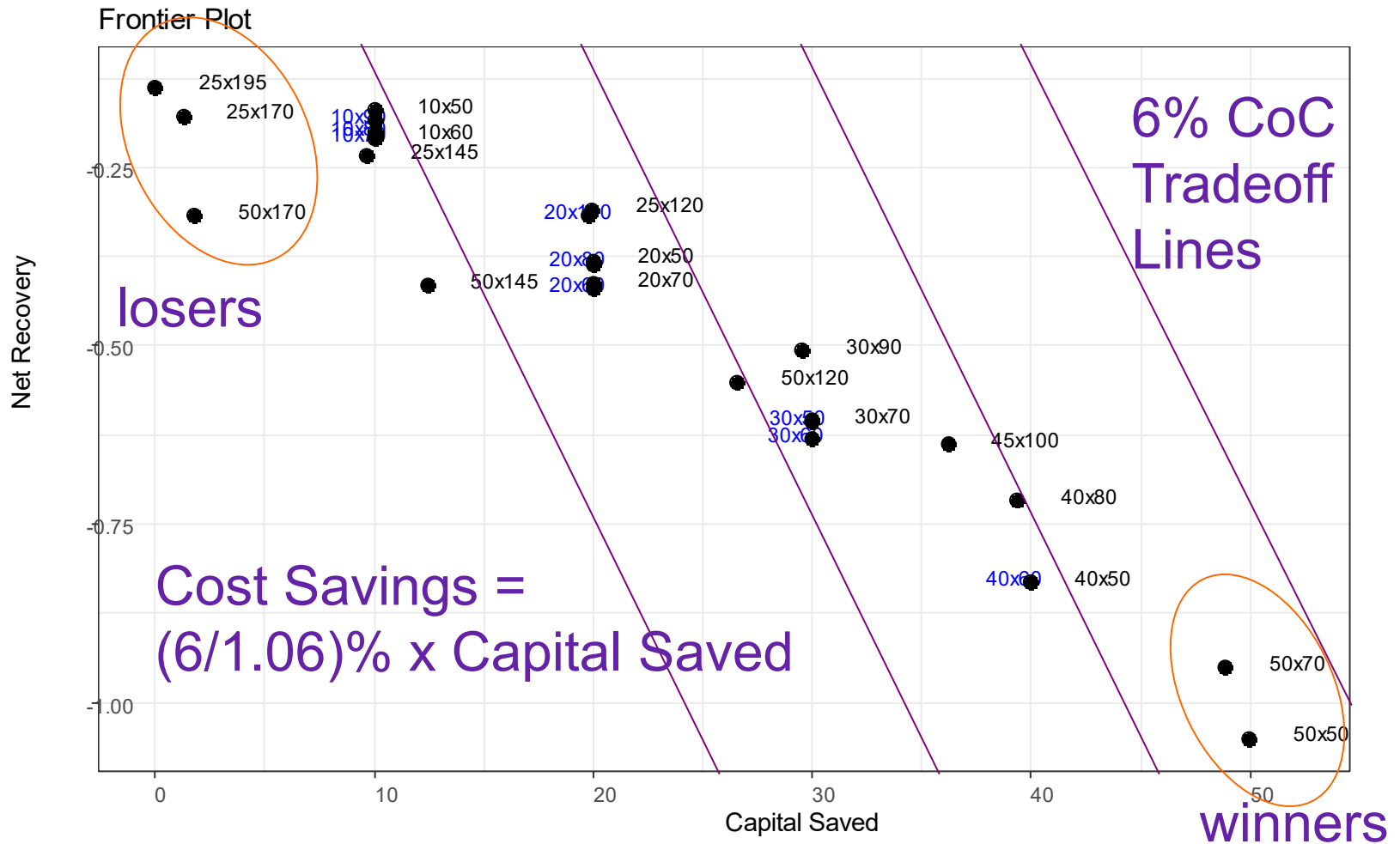
$$P = v E[X \wedge a] + d a$$

- Funded with capital and reinsurance

$$P_{Re} = v E[X_n \wedge a_n] + d a_n + \pi$$

- Difference in funding costs $P - P_{Re}$
= $v(E[X \wedge a] - E[X_n \wedge a_n]) + d \Delta a - \pi$
 $\approx v E[X_c] + d \Delta a - \pi$ ←

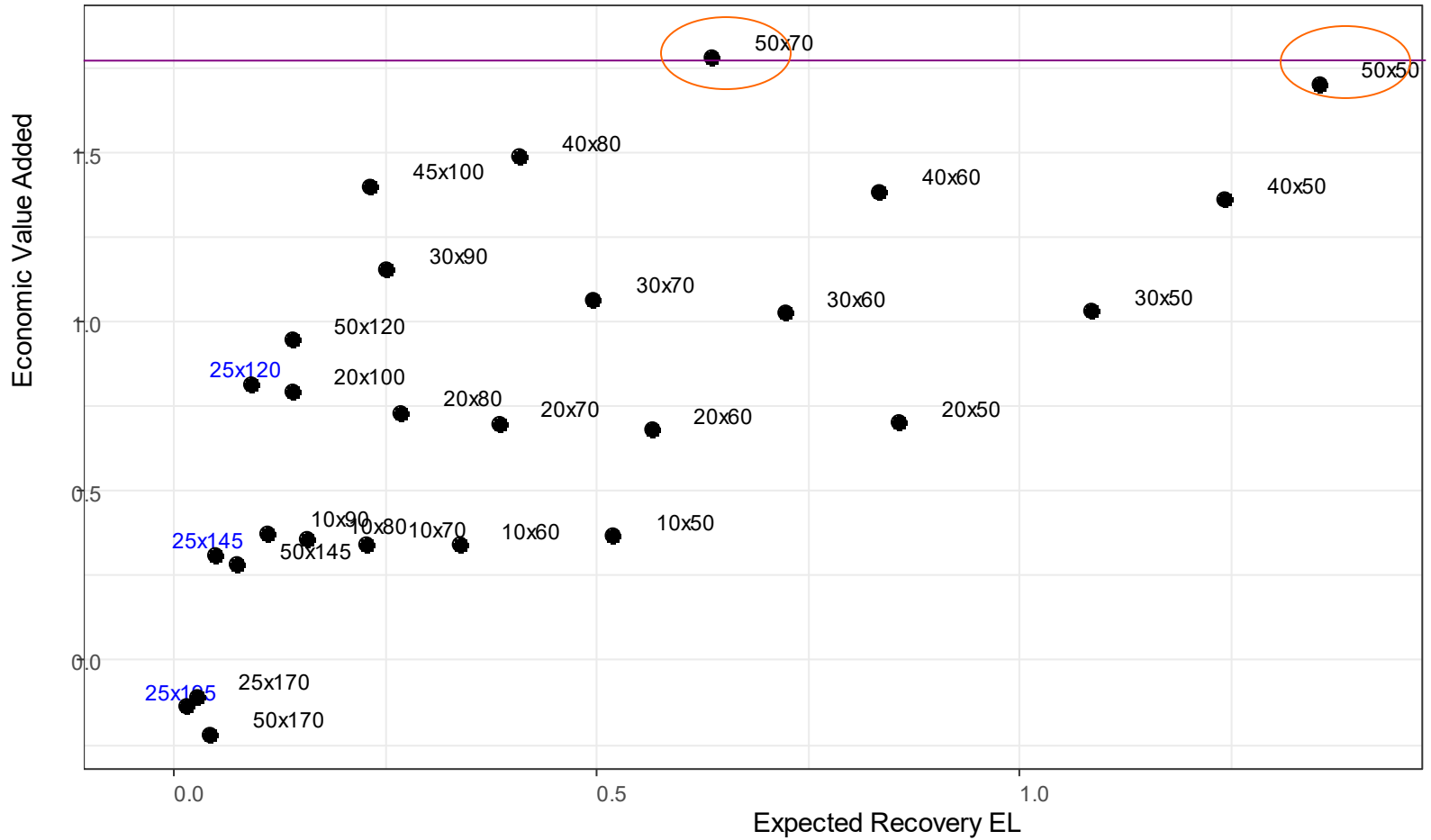
Evaluating Reinsurance with CCoC



CCoC Valuation

1st Place Winner

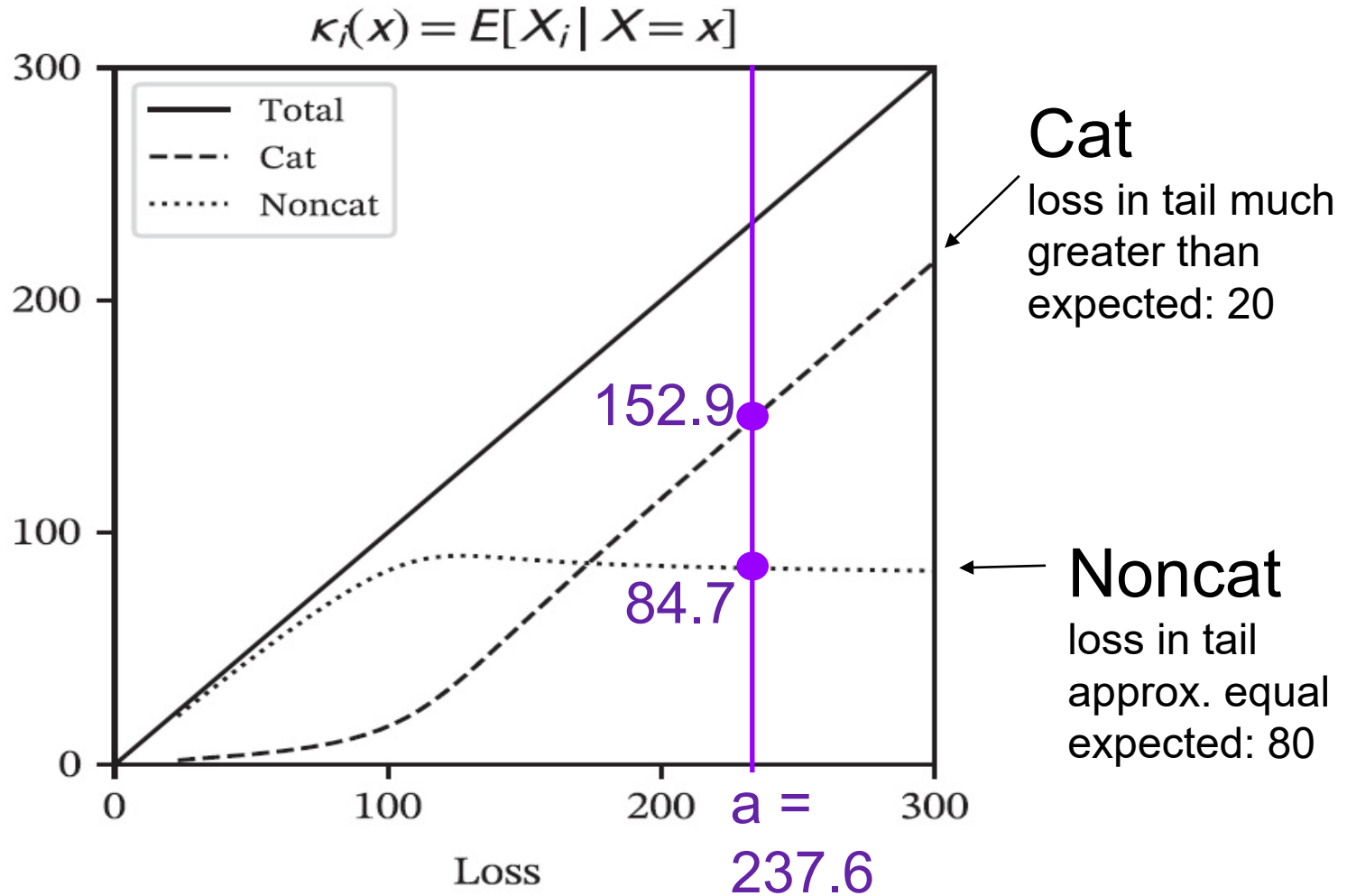
Close 2nd



Evaluating Lines of Business

- $X = w_{\text{cat}} X_{\text{cat}} + w_{\text{nc}} X_{\text{nc}}$ *yeah, yeah, not really*
 $w_{\text{cat}} = w_{\text{nc}} = 1$
- $a = \text{VaR}_{0.998}(X)$
- $\partial a / \partial w_{\text{cat}} = E[X_{\text{cat}} | X = a]$ *marginal asset = co-VaR*
- $P = v \text{EL} + d a$
- $M = P - \text{EL}$
- $\partial M / \partial w_{\text{cat}} = d (E[X_{\text{cat}} | X = a] - E[X_{\text{cat}}])$

Co-VaR, a.k.a Kappa Function



Let's apply to our LOBs!

LOB	Con- ditional EL	Uncon- ditional EL	Shared Liability	Allocation by CCoC Method	
				Profit Margin	Share
Non- Cat	84.7	80	4.7	0.26	3%
Cat	152.9	20	132.9	7.53	97%
Total	237.6	100	137.6	7.79	100%

Is this reasonable? Many people don't think so.

LOB	EL	CCoC Margin	90th %ile	σ^2
Non-Cat	80	0.26 <i>3%</i>	+16 <i>62%</i>	144 <i>26%</i>
Cat	20	7.53 <i>97%</i>	+21 <i>81%</i>	400 <i>74%</i>
Total	100	7.79 <i>100%</i>	+26 <i>100%</i>	544 <i>100%</i>

What's Going On Here

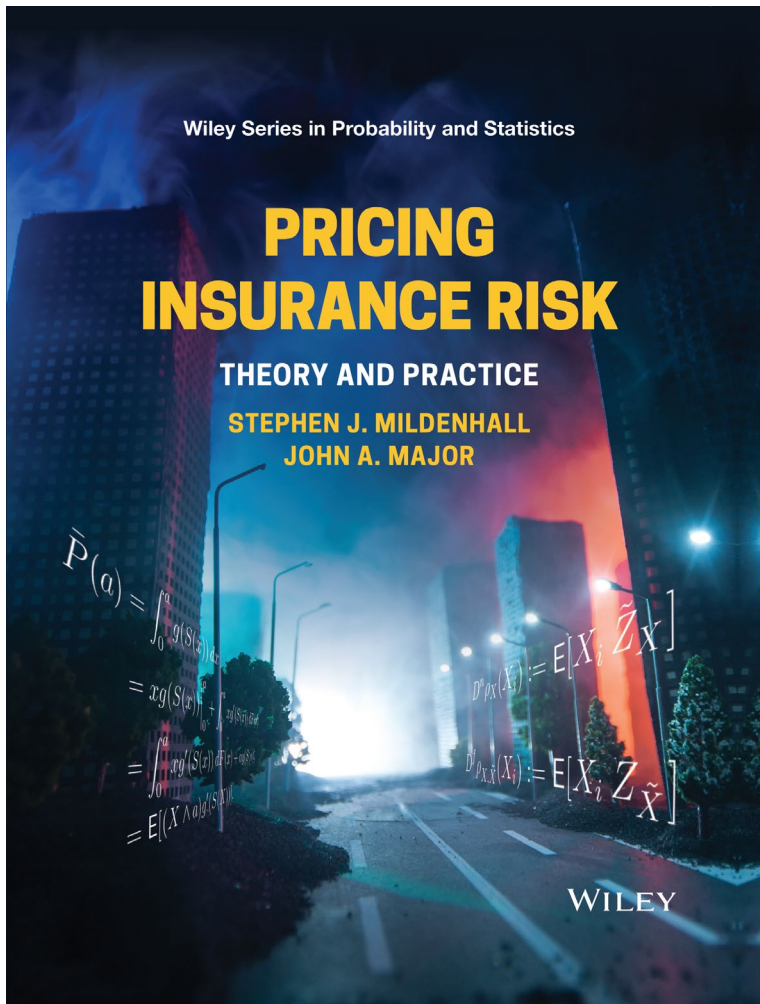
- $P = v EL + d a$
- Agent is
 - Risk neutral v of the time: $P=EL$
 - Doom and gloom d of the time: $P=a$
- CCoC only sensitive to mean + extreme

A Deeper Critique of CCoC

- Capital has a **range of costs**
 - Bonds: credit yield curve
 - Cat bonds at different attachments

- “One return to rule them all” ???
 - Same ROE
 - All LOBs
 - gross & net

Decisions Future? Spectral Risk Measures



15

Modern Price Allocation Practice

In this chapter, we show how to apply the natural allocations and unit funding analysis developed in Chapter 14 to discrete data. We apply the formulas to an adjusted version of Simple Discrete Example introduced in Section 2.4.1. We illustrate how the auxiliary functions κ , α , and β can be used to diagnose risk characteristics of the three Case Studies. We compute the lifted natural allocations for each Case Study. As always, the reader is encouraged to replicate all the calculations.

15.1 Applying the Natural Allocations to Discrete Random Variables

In this section, we show how to compute the various natural allocations of $\rho(X)$ to X_i as part of X by extending the algorithm in Chapter 11 that computes $\rho(X)$. We work with a multivariate discrete distribution as produced by a simulation or catastrophe model.

15.1.1 Algorithm to Compute the Linear Natural Allocation for Discrete Random Variables

Algorithm Inputs:

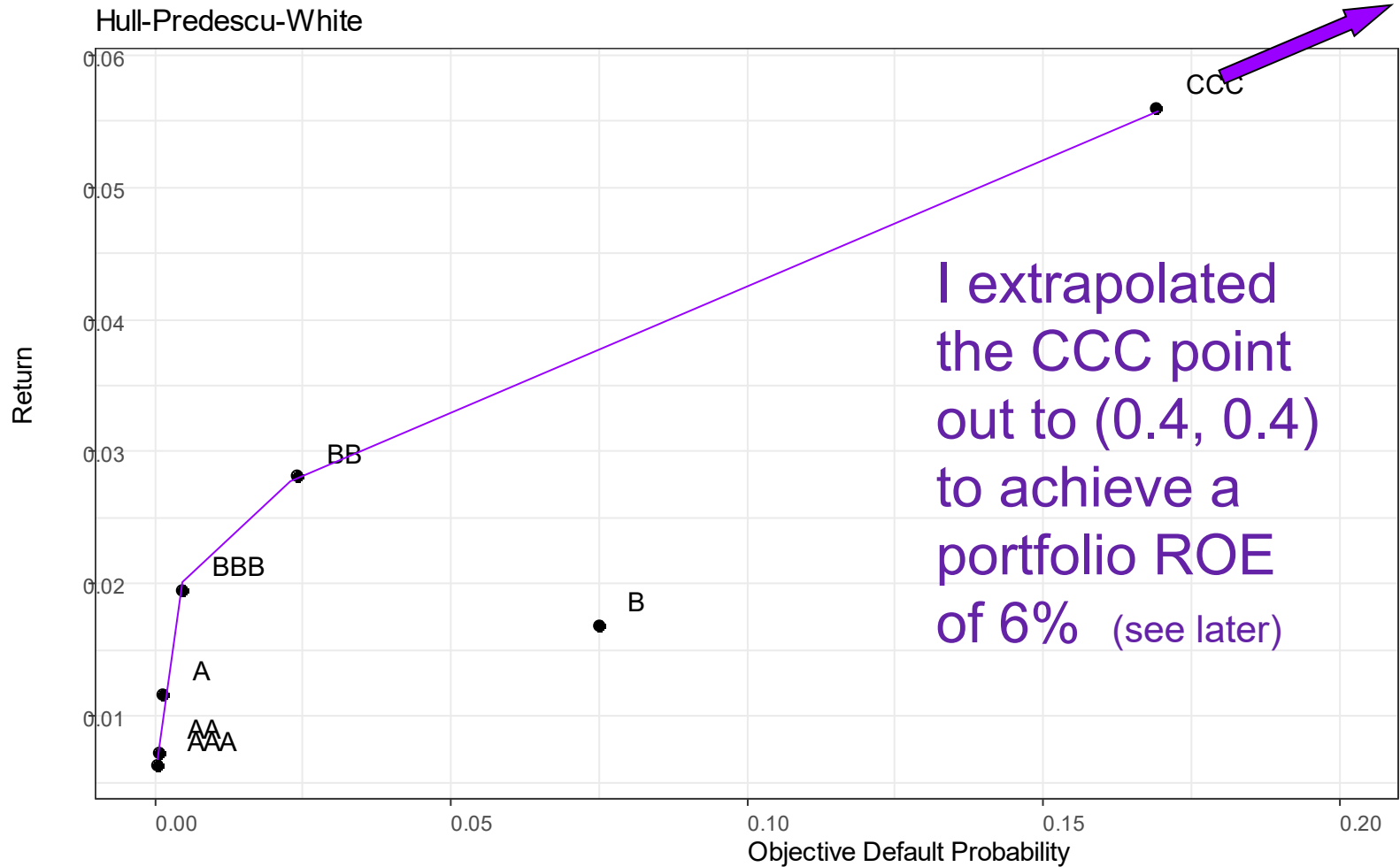
- The outcome values $(X_{1,j}, \dots, X_{m,j})$, $j = 1, \dots, n$, of a discrete m -dimensional multivariate loss random variable. Outcome j occurs with probability p_j . $X_j = \sum_i X_{i,j}$ denotes the total loss for outcome j .
- A spectral risk measure ρ associated with the distortion function g .

Follow these steps to determine $D^{\rho_X}(X_{i,\cdot})$, the natural allocation of $\rho(X)$ to unit i .

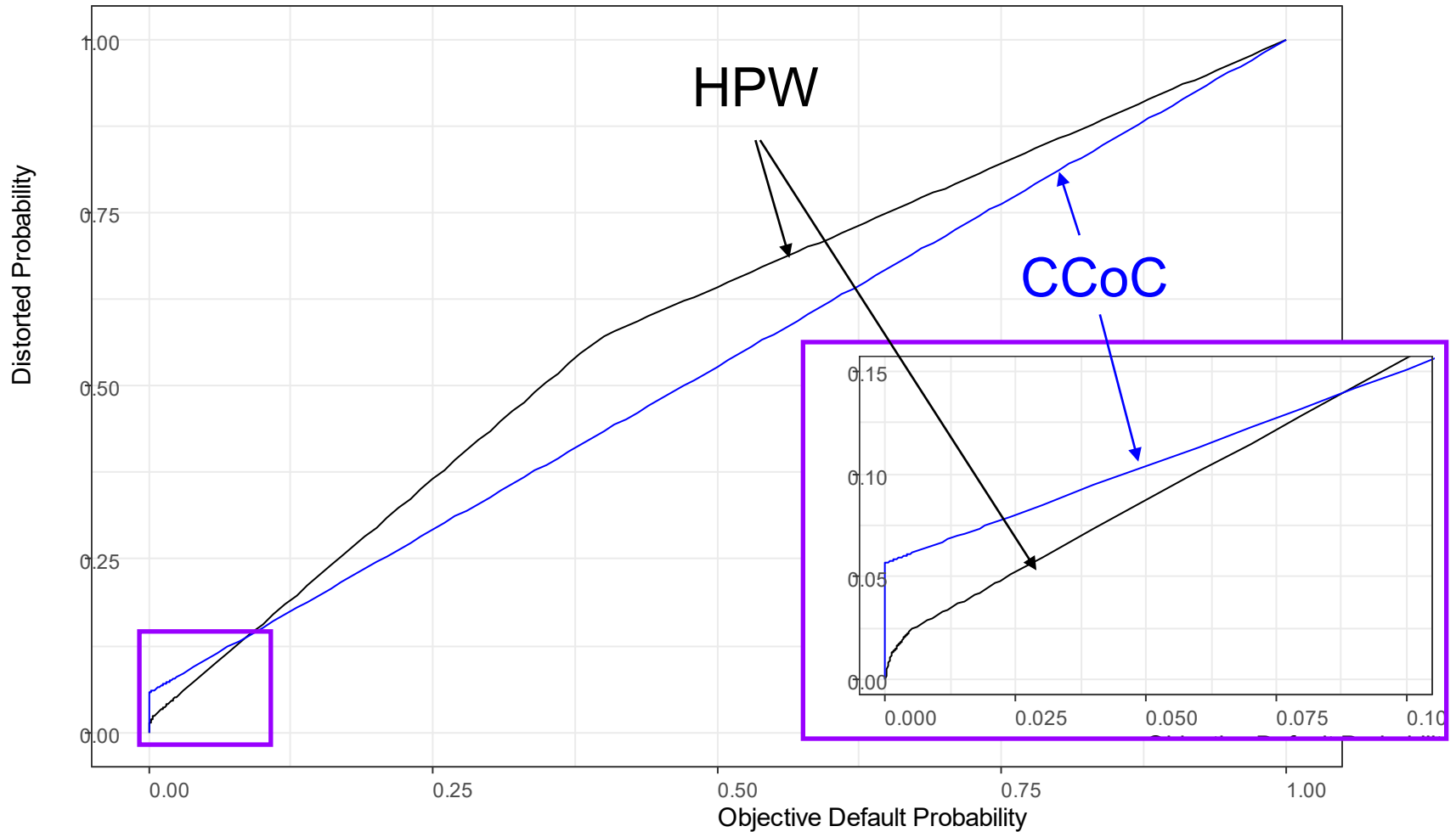
Algorithm Steps

- Pad** the input by adding a zero outcome $X_{1,0} = \dots = X_{m,0} = X_0 = 0$ with probability $p_0 = 0$.
- Sort** events by total outcome X_j into ascending order.

Our Return Model



Our Model Distortion Function vs CCoC



Piecewise Linear $g(s)$

Rating	s	g(s)
0	0.0	0.0
AAA	0.000400	0.006700
A	0.001300	0.012800
BBB	0.004700	0.023800
BB	0.024000	0.050700
CCC	0.396580	0.569995
1	1.0	1.0

So you can reproduce this example on your own.

Applying SRM to *LOBs*

- Simulated scenarios sorted by portfolio loss
- Every scenario j has
 - Probability p_j
 - Exceedance probability s_j
 - Distorted EP $g(s_j)$
 - Distorted probability $\Delta g(s_j)$
- Expected loss for LOB i is $EL_i = \sum_j X_{i,j} p_j$
- Technical Premium $\rho_i = \sum_j X_{i,j} \Delta g(s_j)$
- Margin = $\rho_i - EL_i$

Videos!
[go.guycarp.com/
cas2018](http://go.guycarp.com/cas2018)

SRM Conclusions about LOBs

LOB	EL	CCoC Method		SRM Method	
		Margin	Share	Margin	Share
Non-Cat	80	0.26	3%	2.00	26%
Cat	20	7.53	97%	5.79	74%
Total	100	7.79	100%	7.	100%

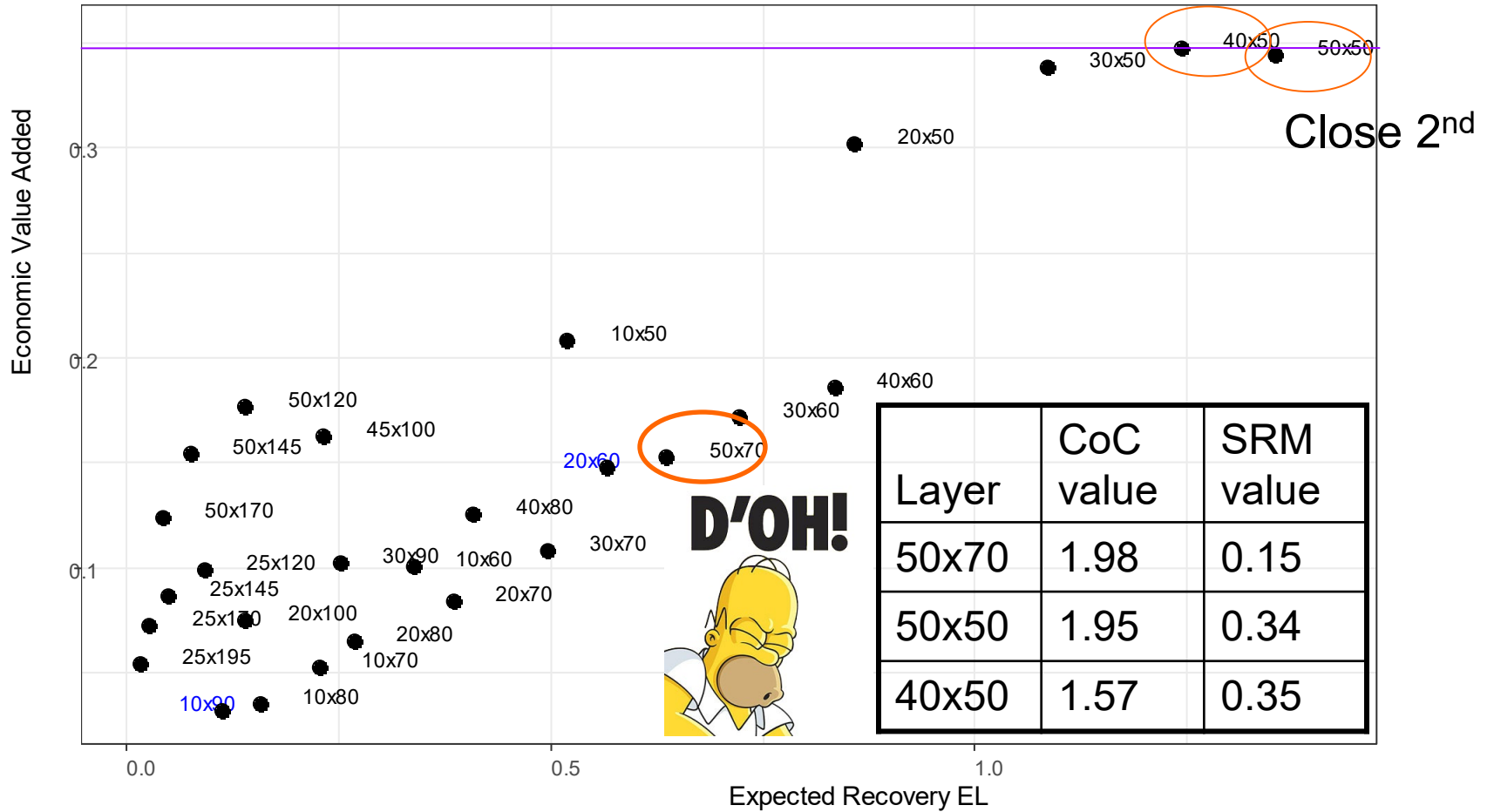
Applying SRM to *Reinsurance*

- Economic Value Added
 - $\rho(X_{\text{gross}}) - \rho(X_{\text{net}})$
 - “A/B method”

- Approximation
 - $\sum_j X_{\text{ceded}, j} \Delta g(s_j)$
 - “Allocate gross”
 - Technically, “linear allocation” $D^n \rho_X(X_c)$

HPW SRM Valuation

1st Place



Takeaways

- Past: Efficient frontiers ... meh
- Present: CCoC ... extreme
- Future: SRMs easy^[1]
 - a. Variable capital cost
 - b. Risk-adjusted probabilities

[1] Terms and conditions apply